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## TIME VALUE OF MONEY

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## Introduction into the Time Value of Money

- ▶ Financial markets exist to match borrowers of funds with lenders of funds.
  - Banks and stockbrokers act as financial intermediaries
  - Interest rates act to clear the market
  - So dollars being lent equals dollars being borrowed



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## Time Value of Money

- ▶ Finance is concerned with money and the price of money is the interest rate
- ▶ Interest rates serve to match supply and demand
- ▶ Always consider the time value of money when evaluating potential investments or projects



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## A dollar today is worth more than a dollar in one year from now...

Think of it as a time line: \$100 earning 8% interest

Year	Beg Value	Interest (8%)	Future Value
1	\$100	\$8	\$108
2	\$108	\$8.64	\$116.64
3	\$116.64	\$9.33	\$125.97

- ▶ The idea of compounding comes into play



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## Present Value

And Vice Versa –

a dollar two years from now is worth less than a dollar today (because the one today could be invested and earn interest)



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## Present Value

- ▶ Mirror image of future value
- ▶ If you are to receive money in X years, how much is that worth today? (Assuming some interest or discount rate)
- ▶ Earlier, said \$100 invested at 8% interest is worth \$125.97 at the end of 3 years (FV). The other side of this is that the PV of \$125.97 at the end of 3 years is \$100.



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## General PV Formula

$$PV = C_0 + \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_n}{(1+r)^n}$$

- ▶  $C_0$  = Cashflow period zero (there may or may not be one)
- ▶  $C_1 \dots n$  = cashflow periods 1 thru n
- ▶  $n$  = last year there is a cashflow
- ▶  $r$  = discount rate (interest rate)



## Present Value Example

You expect to sell your firm for \$35 million in 4 years. If you assume a cost of capital (discount rate) of 15%, what price could you take now and still come out even?



## Present Value Example

$$PV = CF(\text{period 4})/(1+r)^4$$

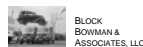
$$35/(1.15)^4 = 35/1.747 = \$20 \text{ million}$$

$$\text{Check: } 20 \cdot 1.15 \cdot 1.15 \cdot 1.15 \cdot 1.15 = 34.98 \text{ million}$$



## Present Value

- ▶ In order to maximize shareholder wealth, a firm must evaluate alternatives and make financing decisions based on the net present value of the project or investment
- ▶ Example: take out a 2 year \$2000 loan at 8% annual interest and invest the \$2000 in a 2 year bond which yields an annual interest of 12%
  - Make a net 2% annual interest on \$2000



## Applying PV to Certain Patterns of Cashflows

1. Perpetuity:  $PV = CF / r$ 
  - $r$  = Rate of Return
  - Perpetuity is an infinite string of terms -- geometric series, although infinite string of terms, it has a finite sum -- use calculus to find above equation
2. Growing Perpetuity:  $PV = CF / (r - g)$ 
  - $g$  = Growth Rate



## Future Value

- ▶ The future value can be calculated for a single amount
- ▶  $FV_n = X_0(1+R)^n$ 
  - $n$  = period
  - $X_0$  = initial amount (period 0) = present value
  - $R$  = interest rate or growth rate or cost of funds
- ▶ Example: 100\$ today is worth what at end of year 2 and year 3 at 8% interest:



## FV example

$$FV_n = X_0(1+R)^n$$

$$FV_{YR 2} = 100(1.08)^2 = 116.64$$

$$FV_{YR 3} = 100(1.08)^3 = 125.97$$



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## Increasing Future Values

► Future values are greater when

- Discount (interest rate) is greater
- The number of periods is greater
- Compounds more frequently



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## Future Value with Compounding

- Now, what if interest is paid more often than once per year?

$$FV_n = X_0 (1+(R/m))^{mn}$$

m = # times per year compounded

n=period

X<sub>0</sub>= initial amount (period 0) = present value

R = interest rate or growth rate or cost of funds



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## Determining Future Values (Example)

► A firm has earnings of \$200,000 and these are expected to grow by 15% per year.

- What will the earnings be at the end of 5 years?

$$200,000 \times (1+.15)^5 = 402,271$$

- Now, what if interest is paid more often than once per year?

$$FV_n = X_0 (1+(R/m))^{mn}$$

m = # times per year compounded



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## Determining Future Values (Example)

- What will \$100 invested now be worth at the end of 3 years if it earns 8% interest rate and compounds semi-annually? (m=2)

$$FV_3 = 100 [ (1+(.08/2)) ]^6 = 126.53$$

(vs 125.97 annual compounding)



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## Determining Future Values (Example)

► As the compounding interval shortens

- FV rises but at a decreasing rate
- Effective interest rate increases

Compounding	Future Value
Annual	\$ 125.97
Semiannual	\$ 126.53
Quarterly	\$ 126.82
Monthly	\$ 127.02
Continuously	\$ 127.12



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## Effective Interest Rate

- ▶ The interest rate actually received (or paid)
  - The interest received (paid) during one year on an investment / the initial investment
- ▶ Example: \$100 earns 8% compounded semiannually. What is its effective interest rate?
  - End of year have  $FV = 100(1+.04)^2 = 108.16$
  - Made \$8.16 in interest
  - Effective interest =  $8.16 / 100 = 8.16\%$



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## Future Values of Annuities

- ▶ Generally assume that payments occur at the end of the period with annuities
  - \$1000 at end of 1<sup>st</sup> period grows throughout years 2 and 3
  - \$1000 at end of 2<sup>nd</sup> period grows throughout year 3
  - \$1000 at end of 3<sup>rd</sup> period doesn't grow



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## Future Value of Annuities (Using Equation)

- ▶  $FV \text{ annuity} = X [((1+R)^n - 1)/R]$

X = amount of the cashflow

R = interest rate

n = number of periods

3 years, 1000 per year, 15% rate

- ▶  $1000[((1.15^3)-1)/0.15] =$   
 $1000[(1.52-1)/0.15] =$   
\$3,472.5



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## Present Value

- ▶ PV of a lump sum =  $Xn/(1+R)^n$ 
  - Xn = \$ get at the end
  - R = discount rate = opportunity cost of funds
  - n = number of years
- ▶ Note: this formula is the reciprocal of the FV formula



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## Present Value (Using Equation)

- ▶ Example: Have the choice of receiving \$500,000 at the end of 3 years or \$350,000 today. Which would you choose if the discount rate is 15%?
  - Need to determine what \$500,000 at the end of 3 years is worth today in order to compare

$$PV = 500,000/(1.15)^3 = 500,000/1.5209 = \$ 328,758$$



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## Present Value of Annuities

- ▶ Example: You are contemplating investing in a project which will bring in \$5000 at the end of each year over the next 3 years.
  - What is the maximum you should invest in this project?
  - The firm's opportunity cost of funds is 14%
- ▶ Primary concept is that one should invest in an opportunity or project if the expected return exceeds the cost of funds to the firm- *even if the firm or individual is not actually borrowing the funds*



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## Present Value of Annuities

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- ▶ Using an equation

$$PV = CF_1/(1+r)^1 + CF_2/(1+r)^2 + CF_3/(1+r)^3 \dots + CF_n / (1+r)^n$$

n= number of periods

CF<sub>n</sub> = Cashflow period n

$$PV = 5000 / 1.14 + 5000 / (1.14^2) + 5000 / (1.14^3) \\ = 4386 + 3847 + 3375 = \$11,608$$



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## Effective Rate of Interest

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$$(1 + (r / m))^m - 1$$

m= # times compounded per year

- ▶ Example: Pays 10% annual rate, compounds 2x per year

$$(1 + .05)^5 - 1 = .103 = 10.3\%$$



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## Brainstorm Applications for PV in a New (or Existing) Firm



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## Present Value Applications

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- ▶ Pricing equity for investors
- ▶ Determining which projects to invest in
- ▶ Determining whether to raise capital for a project



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## Determining Projects (Capital Building)

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- ▶ Investment rules for capital budgeting decisions:
  - Some not theoretically correct- the NPV is the correct method

  1. Net Present Value
  2. Payback Period
  3. Regular IRR (Internal Rate of Return)
  4. Profitability Index



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## Net Present Value Method

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- ▶ Same discounted cash flow technique as present value, but subtracts out the initial investment

$$NPV = PV \text{ inflows} - PV \text{ outflows}$$



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## Interpreting NPV

- ▶ Project can have:
  - Negative NPV
  - Positive NPV
  - Zero NPV
- ▶ What does this mean?
- ▶ What would you invest in a negative or zero NPV project?



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- ▶ Example: You have opportunity to invest in a project which requires an upfront capital investment of \$18,000 and will return to you \$5,600 per year for the next 5 years. If your minimum required return is 12%, should you invest?

$$\text{NPV} = (-\$18,000) + 20,187 \text{ (PV of 5,600 for 5 yrs)} = \$2,187$$

$$\text{PV} = 5,600/(1.12)^1 + 5,600/(1.12)^2 + 5,600/(1.12^3) + 5,600/(1.12^4) + 5,600/(1.12^5) = 20,186$$



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## What if not annual??

- ▶ If payments or cashflows not annual:
  1. multiply # of years \* # times occur per year
  2. divide interest rate by the # of times occur per year



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## Payback Method

- ▶ Number of years required to recover the initial investment.
  - Sometimes compared to a defined hurdle period which the firm has defined.
- ▶ Ex. Firm has opportunity to invest in a project which will cost \$10,000 upfront and will generate net cash flows of \$2,000 per year for 8 years. What is the payback period for this project?
- ▶ Payback period :  $0 = -10,000 + x(2000)$   
 $X = 5 \text{ years}$
- ▶ If net cashflows were \$3,000 per year = 3.33 years



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What are the problems with a payback period??



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## Problems with Payback

1. How determine cutoff period – basically arbitrary
2. Ignores cash flows after the payback period
  - Project 1 cashflows = -6,000, 3000, 3000, 3000,30
  - Project 2 cashflows = -6,000, 3000, 3000, 10,000, 200,000
3. Treats all cashflows before the payback period equally (ie 3,000 in year 2 receives same weight as 3,000 in year 3)

Inaccurate, but seat of pants and simple



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## Internal Rate of Return

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- ▶ Good method, close to the npv concept.
- ▶ Looking at the yield on investment
- ▶ IRR = the interest rate which equates the cash outflows of an investment with subsequent cash inflows
- ▶ IRR is compared to firms cost of capital



## IRR

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- ▶ IRR = discount rate which makes the npv of the project zero

$$0 = -cfo + cf1/(1+r)^1 + cf2/(1+r)^2 + \dots + cfn / (1+r)^n$$

- ▶ Need to solve through iterations (trial and error)



## Problems with IRR Method

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1. Concept breaks down if have any net cashflows which are negative (beyond initial investment) – then could have more than one IRR
2. Assumes cashflows are reinvested at the IRR



## IRR vs NPV Method

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- ▶ Difference is that the IRR assumes cash inflows are reinvested at IRR
- ▶ NPV assumes invested at your cost of capital
- ▶ NPV method assumes all inflows are reinvested at the discount rate used
- ▶ If the disc. rate being used > the IRR, could see negative NPV for a project which meets IRR accept criteria.
- ▶ Any time using a minimum required rate of return = your discount rate, IRR method and NPV method will result in the same conclusion



## Profitability Index

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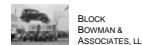
- ▶ Good method - widely used
- ▶  $PI = PV \text{ of cash flows} / \text{initial investment}$
- ▶ Or if multiple cash outflows =  $PV \text{ inflows} / PV \text{ outflows}$
- ▶ Puts projects on comparable terms -- rate of return on investment using PV terms



## Profitability Index (Example)

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- ▶ Investment costs 18,000 upfront then generates 5,600 per year for 5 years.
- ▶ If firm requires a minimum return of 12% (ie cost of capital = 12%) what is the profitability index:  
 $PV \text{ 5,600 for 5 years at 12\%} = 20,187$   
 $PI = 20,187 / 18,000 = 1.12$



## Profitability Index Interpretation

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- ▶ Can directly compare PI values for different projects
- ▶  $PI = 1$  indicates  $NPV = 0$ , indicates just breaking even
- ▶  $PI < 1$  indicates negative NPV, indicates undesirable investment
- ▶ Above 1, the greater the PI, the more desirable



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